COURSE NUMBER: MA1101

COURSE TITLE: Mathematics

COURSE DESCRIPTION:

This is a course designed to prepare students for the study of calculus as well as to introduce and give them a facility with the concepts of differentiation necessary for a better understanding of a variety of technology courses.

PREREQUISITES: Successful completion of either Mathematics MA1700, MA1100, HS Advanced Mathematics 3200**,** or a minimum grade of 70% in HS Academic Mathematics 3201

CO-REQUISITES: None

CREDIT VALUE: Five (5)

COURSE HOURS PER WEEK: Five (5)

LAB HOURS PER WEEK: Zero (0)

SUGGESTED TEXT:

Washington, A. J. (2010). *Basic technical mathematics with calculus. SI version with SSM and MyMathLab/MyStatLab Valuepack access card* (9th ed.). Pearson Education Canada; and Washington, A.J. (2009). *Student solutions manual for basic technical mathematics with calculus* (9th. ed.). New York: Addison-Wesley. ISBN-10:0133077128; ISBN-13: 9780133077124 (Package created by Pearson Canada)

LEARNING RESOURCES: To be determined by instructor

MAJOR TOPICS:

1.0 Complex Numbers
2.0 Functions
3.0 Analytic Geometry
4.0 Trigonometry
5.0 The Derivative
6.0 Matrices
7.0 Curve Sketching

LEARNING OBJECTIVES:

The expected learning outcomes are that the learner will be able to:

1.0 Complex Numbers

1.1 Basic Concepts
- 1.1.1 Define the imaginary unit, $j = \sqrt{-1}$
- 1.1.2 Evaluate the square root of a negative number
- 1.1.3 Simplify expressions involving j^n, where n is a whole number
- 1.1.4 Write any number as a complex number in rectangular form, $a + bj$
- 1.1.5 Solve linear equations involving complex numbers
- 1.1.6 Find the complex conjugate of any complex number, in rectangular form

1.2 Basic Operations
- 1.2.1 Add, subtract, multiply and divide complex numbers in rectangular form
- 1.2.2 Graphically represent any complex number and its associated vector
- 1.2.3 Use vectors to add and subtract complex numbers

1.3 Polar Form
- 1.3.1 Write complex numbers in standard polar form
- 1.3.2 Convert complex numbers from rectangular form to polar form and vice versa
- 1.3.3 Multiply and divide complex numbers in polar forms

1.4 Exponential Form and DeMoivre's Theorem
- 1.4.1 Write complex numbers in exponential form
- 1.4.2 Convert complex numbers from exponential form to both polar and rectangular forms and vice versa
- 1.4.3 Multiply and divide complex numbers in exponential form
- 1.4.4 Find powers and roots of complex numbers using DeMoivre's Theorem
- 1.4.5 Use scientific or graphing calculators to perform complex number operations in any form

2.0 Functions

2.1 Functions of a Single Variable: Definition
- 2.1.1 State the definition of a single-variable function

2.2 Equations, Graphs, Table of Values
- 2.2.1 Identify functions presented in equation form, graphically, or as a table of values
- 2.2.2 Use the vertical line test to distinguish the graph of a function from the graph of a relation

2.3 Dependent and Independent Variables

2.3.1 State the dependent and independent variables, given a function represented as an equation, graph or table of values

2.4 Domain and Range
- 2.4.1 Define the terms "domain" and "range"
- 2.4.2 State the domain and range for any given function

2.5 Functional Notation
- 2.5.1 Explain the functional notation $f(x)$
- 2.5.2 Determine the value of $f(a)$, where a is a real number, given a function in equation form
- 2.5.3 Read the value of $f(a)$ from a graph given that $x = a$

2.6 Relating Graphs and Functional Equations
- 2.6.1 Plot the graph of a linear function of the form $f(x) = ax + b$ using any three points
- 2.6.2 Read ordered pairs of values from a graph to satisfy a given equation and verify the result
- 2.6.3 Explain any small discrepancies in the results above

2.7 Writing Functional Statements from Word Problems
- 2.7.1 Translate word problems into functional equations

2.8 Some Useful Graphical Shapes
- 2.8.1 Use a calculator to develop a table of values and make an accurate graph of each of the following basic expressions: $y = ax^2$; $x = ay^2$; $y = ax^3$
- 2.8.2 Draw an accurate, labeled sketch of the three expressions above without having to first construct a table of values

2.9 Composite Functions
- 2.9.1 Write the expressions for the composite functions $f\left(g(x)\right)$ and $g\left(f(x)\right)$ given $f(x)$ and $g(x)$
- 2.9.2 Evaluate $f\left(g(x)\right)$ and $g\left(f(x)\right)$ given a value for x

2.10 Inverse of a Function
- 2.10.1 Define the inverse of a function
- 2.10.2 Use test for inverse functions: $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$ to verify an inverse
- 2.10.3 Define a one-to-one function
- 2.10.4 Discuss the horizontal line test
- 2.10.5 Determine the inverse of a one-to-one function
- 2.10.6 Graph a function and its inverse on the same graph
- 2.10.7 Discuss examples of modified functions which have inverses such as $y = x^2$, $x \geq 0$

2.11 Reflections

2.11.1 State how to create a reflection of a function with respect to $x - axis\ or\ y - axis$

2.11.2 Write a new expression that creates a reflection of a given expression on the x-axis, or y-axis

2.12 Symmetry

2.12.1 Describe, and illustrate graphically, the relationship between:

$$y = f(x) \ and \ y = -f(x)$$
$$y = f(x) \ and \ y = f(-x)$$
$$y = f(x) \ and \ (-y) = f(-x)$$

2.12.2 Identify functions that are symmetric with respect to the x-axis, y-axis or the origin

2.12.3 Use symmetry to assist with sketching graphs

2.13 Translations

2.13.1 Describe the effect on a graph when x is replaced by $x - a$

2.13.2 Describe the effect on a graph when y is replaced by $y - b$

2.13.3 Sketch the following: $y - b = (x - a)^2 \ or \ x - a = (y - b)^2$

2.13.4 Write the new equation for a given graph after a specified vertical and horizontal translation

3.0 Analytic Geometry

3.1 Review of Conic Sections: Parabola, Circle, Ellipse, and Hyperbola

3.1.1 Identify the standard form equations of the parabola with vertex at (0,0)

3.1.2 Determine the focal point and directrix of the parabola from its standard form equation

3.1.3 Identify the standard form equations of the circle, ellipse, and hyperbola with center at (0,0)

3.1.4 Determine the important features of the curves listed above from their standard form equations

3.2 Conic Sections in Off-Center Positions (Axes parallel to $x \ or \ y \ axis$)

3.2.1 Using the principle of functional translation, write the equations for a parabola with vertex at (h, k)

3.2.2 Using the principle of functional translation, write the equations for a circle, ellipse, or hyperbola with center at (h, k)

3.2.3 Determine the standard form equation for any conic section given its graph

3.2.4 Sketch the graph of any conic section given its standard form equation

3.2.5 Determine the important features of off-center conic sections from the standard form equations

3.3 Applications of Analytic Geometry

3.3.1 Place a figure in the $x - y \ plane$ so that the appropriate conic section equation can be found

3.3.2 Determine the standard form equation of the conic section which describes all or part of the figure
3.3.3 Solve applied problems using the equations found by the previous two steps

3.4 The General Second Degree Equation
3.4.1 State the general second degree equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

3.4.2 State the conditions on the coefficients which identify the equation as a parabola, circle, ellipse, or hyperbola
3.4.3 Identify a conic section given its equation in the general second degree form
3.4.4 Find the standard form equation of a conic section from the second degree equation using the method of completing the square

4.0 Trigonometry

4.1 Inverse Trigonometric Functions
4.1.1 Evaluate inverse trigonometric functions and state two possible values for the angle in the range $0 < \theta < 360^o or\ 0 < \theta < 2\pi$
4.1.2 Select the single value for a given inverse trigonometric function which satisfies a particular given situation

4.2 Trigonometric Identities
4.2.1 Use the basic trigonometric relationships, the sum and difference formulas, half-angle and double angle formulas to prove given trigonometric identities
4.2.2 Use the double angle formulas or sum and difference formulas to simplify trigonometric expressions

4.3 Graphs and Analysis of Trigonometric Functions
4.3.1 Sketch the graphs of the six trigonometric functions
4.3.2 Determine and state the amplitude, period, frequency, phase angle and phase shift for a sinusoidal function expressed in the following forms:

$$y = a\ sin\ (bx + c)$$
$$y = a\ cos\ (bx + c)$$

4.3.3 Use the information above to sketch the graph of any sine or cosine
4.3.4 Given a graph of a sine or cosine function, determine the equation for the function

5.0 The Derivative

5.1 Limit of a Function
- 5.1.1 Define "limit of a function"

5.2 Conditions for Existence of a Limit
- 5.2.1 Distinguish between "left-hand-limit" and "right-hand-limit"
- 5.2.2 State two conditions for the existence of any limit

5.3 Limit of a Function when $x \to a \ \ or \ \ x \to \pm \infty$
- 5.3.1 Evaluate limit of a function when $x \to a$ if the limit exists
- 5.3.2 Evaluate limits of a function when $x \to \pm \infty$

5.4 Slope of a Straight Line
- 5.4.1 Define "slope" as related to a straight line
- 5.4.2 Use the formula $m = \Delta y/\Delta x$ to calculate the slope of a straight line when the graph is given
- 5.4.3 State that $\Delta y/\Delta x$ has the same value as "m" in the equation $y = mx + b$

5.5 Slope of a Curve
- 5.5.1 Calculate the "average slope" of a curve between any two points on the curve by finding the slope of a straight line joining the two points
- 5.5.2 Define "slope of a curve at a point" as the slope of a tangent line at the point
- 5.5.3 Explain why $\lim \Delta y/\Delta x, as\ \Delta x \to 0$ represents the slope of a curve at a point

5.6 The Derivative - Graphical Definition
- 5.6.1 Define "derivative" graphically as the slope of a curve at a point
- 5.6.2 Represent the derivative by the symbols $dy/dx, y', or\ f'(x)$
- 5.6.3 Read dy/dx as "the derivative of y with respect to x, y' as "$y - prime$" and $f'(x)$ as "f$-prime\ of\ x$"
- 5.6.4 Recognize dy/dx as an abbreviation for $\lim \Delta y/\Delta x, as\ \Delta x \to 0$
- 5.6.5 Describe "differentiation" as the process of finding a derivative
- 5.6.6 Use the "first-principles method" (also called the "delta process") to calculate the derivatives of simple functions of the form $y = mx + b\ and\ y = ax^2 + bx + c$
- 5.6.7 Derivative of $y = a, a$ is Constant
  - 5.6.7.1 State that the derivative of a constant is zero and be able to explain why
- 5.6.8 Derivative of $y = x^n$ the Basic Power Rule
  - 5.6.8.1 Use the Delta-process to derive the formula for the derivative of $y = x^n$
- 5.6.9 Derivative of $y = ax^n$, a is Constant
  - 5.6.9.1 Differentiate functions of the form $y = ax^n$

5.6.10 Derivatives of Polynomials
5.6.10.1 Differentiate polynomials with any number of terms

5.7 Product Rule
5.7.1 Derive the product rule using the Delta-process
5.7.2 Use the product rule to differentiate functions of the form $y = uv$ where u and v are two polynomials and are functions of x

5.8 Quotient Rule
5.8.1 Derive the quotient rule using the Delta-process
5.8.2 Use the quotient rule to differentiate functions of the $y = u/v$, where $u \; and \; v$ are polynomials and are functions of x

5.9 Chain Rule and General Power Rule
5.9.1 Derive the chain rule using the delta-process and the Binomial Formula
5.9.2 Use the chain rule to differentiate functions of the form $y = f(u)$, where u is a polynomial function of some other variable
5.9.3 Derive the general power rule using the basic power rule and the chain rule
5.9.4 Use the general power rule to differentiate expressions of the form $y\text{-}u^n$ where u is a function of another variable

5.10 Derivatives with Product/Quotient/Chain Rules Combined
5.10.1 Differentiate functions involving more than one of the rules from sub-topics 5.6 to 5.9

5.11 Factoring and Simplification of Expressions
5.11.1 Factor and simplify expressions of the form $a[f(x)]^m\,[g(x)]^{\,n} + b[f(x)]^p\,[g(x)]^q$ where m, n, p, q are rational numbers

5.12 Differentiation of Implicit Functions
5.12.1 Define "implicit function"
5.12.2 Calculate the derivative of a given implicit algebraic function

5.13 Higher-Order Derivatives
5.13.1 State the meaning of "higher-order derivative"
5.13.2 Write higher-order derivatives in any of the three derivative notations
5.13.3 Calculate the nth derivative of a given function

5.14 Applications of Derivatives
5.14.1 Slopes of Curves
5.14.1.1 Use the derivative to find the slope of a given function at a particular point on the graph of that function

5.14.2 Tangents and Normals
5.14.2.1 Use differentiation to find the slope of the tangent line to a

curve at a particular point

5.14.2.2 Use differentiation and the point-slope form $(y - y_1) = m(x - x_1)$ of a straight line to derive the equation for the tangent line to a curve at a specified point

5.14.2.3 State the relationship between the slope of two perpendicular straight lines

5.14.2.4 Derive the equation for the line normal to a curve at a specified point

5.14.3 Velocity and Acceleration

5.14.3.1 State the definition of a derivative in two ways:

5.14.3.1.1 As the slope of a curve at a point

5.14.3.1.2 As an instantaneous rate of change of one variable with respect to another. *Example: ds/dt is the instantaneous rate of change of displacement, s, with respect to time, t, and thus is an expression for instantaneous velocity*

5.14.3.2 Distinguish between $\Delta s/\Delta t$ *and* ds/dt

5.14.3.3 Use differentiation to find the instantaneous velocity when the displacement function is given

5.14.3.4 Use higher-order derivatives to calculate velocity and acceleration when a displacement function is given

5.14.4 General Rates of Change

5.14.4.1 Use differentiation to give an expression for the rate of change of any dependent variable with respect to the independent variable

6.0 Matrices

6.1 Definition and basic features of matrices

6.1.1 Define a matrix as a rectangular array with m rows and n columns

6.1.2 Define the dimension of a matrix as m x n

6.1.3 Define a square matrix as an m x m array

6.1.4 Define the identity matrix, I, as a square matrix with "1's" on principal diagonal and "O's" elsewhere

6.2 Multiply a vector or matrix by a scalar

6.3 Evaluate the determinant of a square matrix using a Graphing Calculator

6.4 Add, subtract, and multiply matrices by hand or by calculator

6.5 Find the inverse of a square matrix

6.5.1 Find the inverse of a 2 x 2 matrix by hand

6.5.2 Find the inverse of a higher order matrix using a graphing calculator

6.6 Solve systems of linear equations using matrix inversions

6.6.1 Solve systems of 2 linear equations using matrix inversion by hand

6.6.2 Solve systems of 3 (or more) linear equations using matrix inversion by calculator

6.7 Solve various applied problems that lead to systems of linear equations. *Examples: number problems, mixture problems, motion problems, mechanics problems, circuit problems, financial problems, etc.*

7.0 Curve Sketching

7.1 Intercepts
- 7.1.1 Find any x-intercepts and y-intercepts for a given function

7.2 Vertical Asymptotes
- 7.2.1 Define the term "asymptote" and find any vertical asymptotes

7.3 Horizontal Asymptotes
- 7.3.1 Use limits to find any horizontal asymptotes

7.4 Range and Domain of a Function
- 7.4.1 Use limits, where necessary, to assist in finding the range of a function

7.5 Behaviour as $x \to \pm\infty$
- 7.5.1 State the behaviour of a function as $x \to \pm\infty$

7.6 Behaviour as Function Approaches Asymptotes
- 7.6.1 State the behaviour of a function as x approaches the asymptote

7.7 Using Derivatives to Locate Important Points or Intervals in a Function
- 7.7.1 Critical Points and Critical Values
  - 7.7.1.1 Define "critical points" in a function or its graph: Maximum, Minimum and Inflection point
  - 7.7.1.2 Define "critical values" of a variable in relation to the three critical points
- 7.7.2 Locating Critical Points with Derivatives
  - 7.7.2.1 Explain why the first derivative of a function is zero at a maximum or minimum point
  - 7.7.2.2 Use the first derivative test to determine the critical values for the maximum and minimum points, and give the coordinates of the points
  - 7.7.2.3 Explain why the second derivative of a function is positive at minimum points and negative at maximum points
  - 7.7.2.4 Use the second derivative test to determine whether a critical point is a maximum or minimum
  - 7.7.2.5 Recognize that an inflection point occurs when the second derivative has a value of zero

7.7.2.6 Use the second derivative to determine the coordinates of any inflection points

7.7.3 Increasing and Decreasing Sections of a Functions

7.7.3.1 Explain what is meant by "increasing" and "decreasing" as related to functions

7.7.3.2 State how the first derivative can show where a function is increasing or decreasing

7.7.3.3 Use the first derivative to find intervals where a function is increasing or decreasing

EVALUATION:

2 Term Tests	40%
Assignments (Take Home or in Class)	10%
Final Examination	50%

DATE DEVELOPED: June 1994 **DATE REVIEWED:** February 2011

REVISION NUMBER: 16 **DATE REVISED:** June 2014

Note to instructor: Check PIRS to ensure this outline is the most current version.