**COURSE NUMBER:** ET2150

**COURSE TITLE:** Advanced Circuit Analysis

**COURSE DESCRIPTION:**

In this course, learners will review techniques of differential equations, first order and second order: integral combinations; growth and decay problems; the analysis and solution of source free RL and RC circuits; driven RL and RC circuits using differential integral calculus; sinusoidal analysis; the concept of phasors, and steady state response. The learner will learn mathematical techniques and apply these to the concepts to analyze and solve differential equations.

Topics include waveform analysis and synthesis, time domain analysis, solution of differential equations using LaPlace transforms, application of LaPlace transforms to solve electric circuits, and derivation of transfer functions. In addition, the following topics will be covered in this course: Fourier expansion of periodic function, even and odd, Fourier analysis of waveforms and their application to electrical signals, and impulse response.

**PREREQUISITES:** MA2100 – Mathematics
ET1151 – Circuit Analysis II or
MP2140 – Circuit Analysis I

**CO-REQUISITES:** None

**CREDIT VALUE:** Five (5)

**COURSE HOURS PER WEEK:** Five (5)

**LAB HOURS PER WEEK:** Zero (0)

**SUGGESTED TEXT:**

Washington, A.J. (2009). *Basic technical mathematics with calculus, SI version* (9th ed.). Toronto, ON: Pearson Education Canada. ISBN 10-013506712X; ISBN 13-9780135067123

Hayt, W.H., Kemmerly, J., & Durbin, S.M. (2012). *Engineering circuit analysis.* McGraw-Hill Science/Engineering/Math. ISBN 10: 0073529575; ISBN 13: 978-0-07-3529578

**LEARNING RESOURCES:** To be determined by instructor

**MAJOR TOPICS:**

1.0 Differential Equations
2.0 Transient Circuits
3.0 Sinusoidal Analysis
4.0 Fourier Transforms
5.0 Fourier Series

**LEARNING OBJECTIVES:**

The expected learning outcomes are that the learner will be able to:

**1.0 Differential Equations**

1.1 Definition
- 1.1.1 Define the term ‘Differential Equation’
- 1.1.2 State examples of differential equations

1.2 Classification: Order, Degree
- 1.2.1 Define “order” as related to a differential equation
- 1.2.2 Define “degree” as related to a differential equation
- 1.2.3 State the order and degree of a given differential equation

1.3 Solutions: Definition, General, Particular, Testing of Given Solution
- 1.3.1 Define “solution” in relation to a differential equation
- 1.3.2 Distinguish between “general solution” and a “particular solution”
- 1.3.3 Verify that a given equation is a solution for a stated differential equation

1.4 Solution Techniques
- 1.4.1 Separation of Variables
  - 1.4.1.1 Apply the technique of separation of variables to solve a given differential equation
  - 1.4.1.2 Use additional available information to determine the particular solution of a given equation after applying this technique

- 1.4.2 Integrable Combinations
  - 1.4.2.1 Use the following differential combinations to obtain the general or particular solution of a given equation:
    - 1.4.2.1.1 $d(xy) = x\,dy + y\,dx$
    - 1.4.2.1.2 $d(x^2 + y^2) = 2x\,dx + 2y\,dy$
    - 1.4.2.1.3 $d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$
    - 1.4.2.1.4 $d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dx}{y^2}$

- 1.4.3 First-Order Linear Equations
  - 1.4.3.1 Identify a given equation as first-order linear by comparing it

with one of the following two forms:

1.4.3.1.1 dy + Py = Q or dx

1.4.3.1.2 dy + Py dx = Q where P and Q are functions of x only or are constants

1.4.3.2 Explain the purpose of the integrating factor in solving such an equation

1.4.3.3 Solve a given first-order differential equation

1.4.4 Homogeneous First-Order Differential Equations

1.4.4.1 State the identifying characteristic of a homogeneous first-order differential equation

1.4.4.2 State the substitutions necessary to solve this type of equation

1.4.4.3 Solve Homogeneous first-order differential equations

1.4.5 Second-Order Differential Equations

1.4.5.1 Solve differential equations of the following types:

1.4.5.1.1 R.H.S. = O

1.4.5.1.2 R.H.S. = f(x)

1.4.5.1.3 Auxiliary equation having real, equal, distinct or complex roots

1.5 Circuit Applications

1.5.1 Set up the differential equation to represent a given series RLC-circuit and solve the resulting equation to obtain an equation for current or voltage in terms of time

1.5.2 Use given values to obtain the particular solution for a given circuit

1.5.3 Find the current or voltage in a given circuit at a particular time

## 2.0 Transient Circuits

2.1 Inductor/Capacitor Current/Voltage Relationships

2.1.1 Write the current /voltage expressions for inductance

2.1.2 Sketch a basic RL circuit and explain relation between voltage and current

2.1.3 Write the current /voltage expressions for capacitance

2.1.4 Sketch a basic RC circuit and explain relation between voltage and current

2.2 Source-Free RL and RC Circuits

2.2.1 Determine a response in source-free RL circuit

2.2.2 Sketch the resulting waveforms for a source free RL circuit

2.2.3 Determine a response in source-free RC circuit

2.2.4 Sketch the resulting waveforms for a source free RC circuit

2.3 Driven RL and RC Circuits

2.3.1 Determine a response in a driven RL circuit

2.3.2 Sketch the resulting waveforms for a driven RL circuit

2.3.3 Determine a response in a driven RC circuit
2.3.4 Sketch the resulting waveforms for a driven RC circuit

## 3.0 Sinusoidal Analysis

3.1 The Phasor Concept
- 3.1.1 Determine Phasor relationships in sinusoidal circuits
- 3.1.2 Convert from a sinusoidal expression to a phasor expression
- 3.1.3 Convert from a phasor expression to a sinusoidal expression

3.2 Steady-State Response
- 3.2.1 Apply techniques of circuit analysis to determine responses in sinusoidal circuits, using phasor concepts

3.3 Definition of Laplace Transform
- 3.3.1 State the integral definition of Laplace Transform
- 3.3.2 State the purpose of Laplace Transforms in relation to differential equations

3.4 Some Properties
- 3.4.1 State the following properties of Laplace Transforms
  - 3.4.1.1 L[af(t)] = aL[f(t)] = a F(s)
  - 3.4.1.2 L[as(t) + bg(t)] = a(L[f(t)]) = a F(s) = b G(s)

3.5 Finding Laplace Transforms
- 3.5.1 Find the Laplace Transform of a given function y = f(t) using the definition
- 3.5.2 Find the Laplace Transform of a given function y = f(t) using the table of Laplace Transforms

3.6 Inverse Laplace Transforms
- 3.6.1 Define the term inverse Laplace Transform
- 3.6.2 Use a given table to find certain inverse Laplace Transforms

3.7 Laplace Transforms of Derivatives
- 3.7.1 Calculate Laplace transforms of first and second derivatives

3.8 Solution of Differential Equations using Laplace Transforms
- 3.8.1 Find the particular solution of a given differential equation given certain initial conditions

3.9 Use of Laplace Transforms in Electrical Circuit Applications
- 3.9.1 Use Laplace transforms to derive equations for the current or voltage in a given RL series circuit
- 3.9.2 Use Laplace transforms to derive equations for the current or voltage in a given RC series circuit

3.9.3 Use Laplace transforms to derive equations for the current or voltage in a given RLC-series circuit
3.9.4 Derive an equation for the transfer function of a circuit using Laplace transforms for initially relaxed RL and RC circuits

**4.0 Fourier Transforms**

4.1 Derivation of Fourier Transform
4.1.1 Derive the Fourier Transform from Fourier Series coefficients

4.2 Singular Functions
4.2.1 Define singular functions
4.2.2 Describe the unit step function u(t)
4.2.3 Describe the unite impulse (delta) function S(t)
4.2.4 Recognize the importance of the unit impulse function in electric system analysis

4.3 Useful properties of Fourier Transforms
4.3.1 Identify the properties of Fourier Transforms

4.4 Impulse Response and Transfer Function of Electrical Systems
4.4.1 Define a Fourier Transfer Function of an electric network
4.4.2 Define impulse response of an electric network
4.4.3 Determine the transfer function of electric circuits

**5.0 Fourier Series**

5.1 Definition
5.1.1 State the general form of a trigonometric Fourier Series for a function having a period of 2p

5.2 Fourier Coefficients
5.2.1 State the integration formulas used to calculate the Fourier coefficients in a trigonometric Fourier Series

5.3 Fourier Expansion of Periodic Functions
5.3.1 Sketch the required number of periods for periodic functions with a period of:
5.3.1.1 2p
5.3.1.2 2L

5.3.2 Identify aN as the average value of the given waveform and be able to determine its value either from the formula or from inspection of the graph
5.3.3 Expand a given periodic function into a trigonometric Fourier Series containing a required number of terms

5.4 Odd and Even Functions
  5.4.1 Identify periodic functions as either odd or even
  5.4.2 Explain the relation between odd or even functions and the form of the corresponding Fourier Series

5.5 Exponential Fourier Series
  5.5.1 Write the general form for the exponential Fourier Series
  5.5.2 State the mathematical relationship between the coefficients of exponential and trigonometric Fourier Series
  5.5.3 Convert a given Fourier Series from trigonometric to exponential form and vice versa
  5.5.4 Derive the exponential Fourier Series for a given function by developing the trigonometric series and converting the result to exponential form

5.6 Fourier Analysis of Waveforms
  5.6.1 Determine the d.c. component of a given waveform
  5.6.2 State the percentage of various harmonics present in a given waveform

5.7 Application to Electrical Signals
  5.7.1 Analyze electrical signals using Fourier Series

**EVALUATION:**

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|---|---|
| Assignments / Activities: | 30% |
| Tests and / or Quizzes: | 30% |
| Final Exam: | 40% |

**DATE DEVELOPED:** February 2011 **DATE REVIEWED:**

**REVISION NUMBER:** 1 **DATE REVISED:** March 2012

*Note to instructor: Check PIRS to ensure this outline is the most current version.*