

COURSE NUMBER: MA2100

COURSE TITLE: Mathematics

COURSE DESCRIPTION:

In this course students will extend their study of topics in differential calculus and will also be introduced to integral calculus. Topics covered will assist students to better understand concepts encountered in other courses.

PREREQUISITES: MA1101 – Mathematics

CO-REQUISITES: None

CREDIT VALUE: Five (5)

COURSE HOURS PER WEEK: Five (5)

LAB HOURS PER WEEK: Zero (0)

SUGGESTED TEXT: To be determined by instructor

LEARNING RESOURCES: To be determined by instructor

MAJOR TOPICS:

- 1.0 Transcendental Functions and Their Derivatives
- 2.0 Applications of the Derivative
- 3.0 Differentials
- 4.0 Introduction to Integration
- 5.0 The Definite Integral
- 6.0 General Techniques of Integration
- 7.0 Selected Applications of Indefinite and Definite Integrals

LEARNING OBJECTIVES:

1.0 Transcendental Functions and Their Derivatives

- 1.1 Derivatives of the trigonometric functions
 - 1.1.1 State the formulas for the derivatives of the six trigonometric functions
 - 1.1.2 Use the above formulas and the differentiation rules from Mathematics 1101 (power, product, quotient and chain rules) to differentiate

trigonometric functions

- 1.2 Derivatives of the inverse trigonometric functions
 - 1.2.1 Derive the formulas for the derivatives of the three inverse trigonometric functions
 - 1.2.2 Use the above formulas to differentiate inverse trigonometric functions
- 1.3 Derivatives of exponential functions
 - 1.3.1 Differentiate functions containing the forms a^u or e^u where u is a function of another variable
 - 1.3.2 State the exponential definitions of the three basic hyperbolic functions, $\sinh(u)$, $\cosh(u)$, and $\tanh(u)$
 - 1.3.3 Derive the formulas for the derivatives of the hyperbolic functions using the exponential definitions
 - 1.3.4 Use the above derived formulas to differentiate hyperbolic functions
 - 1.3.5 Use hyperbolic functions and differentiation to solve simple related problems
- 1.4 Derivatives of Logarithmic Functions
 - 1.4.1 Calculate the derivative of a function of the form $y = \log_a u$ or $y = \log_e u$ where u is a function of x
 - 1.4.2 Apply the properties of logarithms to simplify logarithmic expressions before the derivative is found
 - 1.4.3 Solve simple problems involving logarithmic functions and their derivatives

2.0 Applications of the Derivative

- 2.1 Maximum and Minimum Problems
 - 2.1.1 Solve maximum and minimum problems related to motion, areas, volumes, cost, beam strength, light intensity, electricity
- 2.2 Parametric Equations and Curvilinear Motion
 - 2.2.1 State the meaning of $\arctan x$ and evaluate, using a calculator, for a specific value of x
 - 2.2.2 Use differentiation, and the formulas $v = \sqrt{v_x^2 + v_y^2}$ and $\tan \theta_v = \frac{v_y}{v_x}$ to solve curvilinear motion problems
 - 2.2.3 Use differentiation, and the formulas $a = \sqrt{a_x^2 + a_y^2}$ and $\tan \theta_a = \frac{a_y}{a_x}$ to solve curvilinear motion problems
- 2.3 Basic Electrical Parameters: Charge, Current, and Power
 - 2.3.1 Solve simple electrical problems using the following derivative formulas:
 - 2.3.1.1 $i = dq/dt$

$$2.3.1.2 \quad p = dw/dt$$

2.4 Related Rate Problems

2.4.1 Use differentiation to solve related rate problems involving motion, mechanics, thermal expansion and fluid flow

2.5 Newton's Method

2.5.1 Find the roots of a given equation, correct to any specified number of decimal places, using Newton's Method

3.0 Differentials

3.1 Definition

3.1.1 Define dy and dx for a function $y = f(x)$

3.2 Graphical and geometric interpretations

3.2.1 Compare dy and Δy both graphically and geometrically

3.2.2 Calculate the differential of a given function

3.3 Applications of differentials

3.3.1 Approximation of small amounts of change

3.3.1.1 Use differentials to approximate small changes in a dependent variable such as area, volume, or displacement due to a small change in the independent variable

3.3.2 Effects of small errors in measurement on calculated quantities

3.3.2.1 Use differentials to calculate absolute errors and relative errors due to small errors in measurement

4.0 Introduction to Integration

4.1 Anti-differentiation

4.1.1 Explain the relation between differentiation and integration

4.1.2 Use this relationship with elementary functions to find the function when the differential is known

4.2 The indefinite integral: $\int x^n dx$

4.2.1 State the formula: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

4.2.2 Explain why the "constant of integration", C , is present in the above formula and hence the reason for the term "indefinite" integral

4.2.3 Explain the terms "integral" and "integration"

4.2.4 Use the above formula to integrate simple monomials of the form

x^n , where $n \in \mathbb{R}$ and $n \neq -1$

- 4.2.5 Use differentiation to check the results of using the above formula
- 4.3 Integrals of the form $\int ax^n dx$ where "a" is a constant
- 4.3.1 Integrate monomials of the form ax^n using the property $\int ax^n dx = a \int x^n dx$, where "a" is a constant
- 4.4 Integrals of polynomials
- 4.4.1 Integrate polynomials using the property
- $$\int (u \pm v \pm w \dots) dx = \int u dx \pm \int v dx \pm \int w dx \dots$$
- 4.5 Finding the functional relationship given the derivative or differential
- 4.5.1 Find the particular function or family of functions given information about the derivative or differential
- 4.6 The indefinite integral $\int u^n du$, where "u" is a function of another variable
- 4.6.1 State the formula $\int u^n du = \frac{u^{n+1}}{n+1} + C$
- 4.6.2 State the value of n for which this formula is invalid
- 4.6.3 Use the above formula to integrate compound functions of the form u^n , where u is a function of another variable
- 4.7 Integrals of exponential functions
- 4.7.1 State the formulas $\int e^u du = e^u + C$ and $\int a^u du = \frac{a^u}{\ln a} + C$
- 4.7.2 Use the above formulas to integrate exponential functions
- 4.7.3 Check those formulas using differentiation
- 4.8 Integrals of the form $\int \frac{du}{u}$
- 4.8.1 State the formula $\int \frac{du}{u} = \ln |u| + C$
- 4.8.2 Use the formula to integrate expressions matching this form
- 4.9 Integrals of trigonometric functions
- 4.9.1 State the following formulas for the integrals of trigonometric functions:
- 4.9.1.1 $\int \sin u du = -\cos u + C$
- 4.9.1.2 $\int \cos u du = \sin u + C$
- 4.9.1.3 $\int \tan u du = -\ln(\cos u) + C = \ln(\sec u) + C$
- 4.9.1.4 $\int \cot u du = \ln(\sin u) + C = -\ln(\csc u) + C$
- 4.9.1.5 $\int \sec u du = \ln(\sec u + \tan u) + C$
- 4.9.1.6 $\int \csc u du = \ln(\csc u - \cot u) + C$
- 4.9.1.7 $\int \sec^2 u du = \tan u + C$
- 4.9.1.8 $\int \csc^2 u du = -\cot u + C$

$$4.9.1.9 \quad \int \sec u \tan u \, du = \sec u + C$$

$$4.9.1.10 \quad \int \csc u \cot u \, du = -\csc u + C$$

4.9.2 Use the above formulas to integrate trigonometric functions

4.10 Factoring and Simplification of Expressions

4.10.1 Factor and simplify expressions of the form $a[f(x)]^m [g(x)]^n + b[f(x)]^p [g(x)]^q$ where m, n, p, q are rational numbers

5.0 The Definite Integral

5.1 Area under a curve - definition

5.1.1 Define "area under a curve"

5.2 Approximating areas under curves

5.2.1 Use either the rectangular method or the Trapezoidal Rule to approximate areas under curves

5.3 The Fundamental Theorem of Integral Calculus, and the Definite Integral

5.3.1 State the Fundamental Theorem of Integral Calculus

5.4 Evaluation of definite integrals

5.4.1 Use the Fundamental Theorem to evaluate definite integrals

5.5 Simpson's and Trapezoidal Rules

5.5.1 Use Simpson's Rule and the Trapezoidal Rule to approximate the area under a given curve

5.6 Approximating definite integrals

5.6.1 Approximate the value of a definite integral using the methods of 5.2 or 5.5

5.7 The definite integral and areas under curves

5.7.1 Use the definite integral to calculate areas under curves

5.8 Areas between curves

5.8.1 Use the definite integral to calculate areas bounded by two or more curves

5.9 Areas involving either dx or dy

5.9.1 Calculate bounded areas by integrating with respect to either x or y

5.10 Volumes by Integration: Disk and Cylindrical Shell Methods

5.10.1 Use integration to calculate volumes of solids of revolution using either the "disk" method or the "shell" method

6.0 General Techniques of Integration

6.1 Integration by Parts

- 6.1.1 Use the integration by parts technique to integrate logarithmic and inverse trigonometric functions as well as certain products that are not among the basic forms

6.2 Integration by Trigonometric Substitution

- 6.2.1 Use a suitable trigonometric substitution to integrate expressions containing one of the following radical forms:
 $\sqrt{a^2 + b^2u^2}$, $\sqrt{a^2 - b^2u^2}$, $\sqrt{b^2u^2 - a^2}$

6.3 Integration by Algebraic Substitution

- 6.3.1 Use algebraic substitution to integrate expressions containing the form $ax + b$

6.4 Integration using Partial Fractions

- 6.4.1 Integrate rational fractions by first separating the given fraction into its corresponding partial fractions
- 6.4.2 Find the partial fractions where the original fraction contains either linear or quadratic factors (unique or repeated) in its denominator

6.5 Integration using Tables

- 6.5.1 Select an appropriate formula from a given table of integrals and use it to integrate a given expression

7.0 Selected Applications of Indefinite and Definite Integrals

7.1 Acceleration, Velocity, Displacement

- 7.1.1 Use integration to solve problems relating to acceleration, velocity, displacement

7.2 Electric Current, Charge and Voltage

- 7.2.1 Use integration to solve basic problems relating to electric current, charge, and voltage

EVALUATION:

Term Tests: 40%
Home/In-Class Assignments: 10%
Final Exam: 50%

DATE DEVELOPED: June 1994

DATE REVIEWED: March 2013

REVISION NUMBER: 3

DATE REVISED: May 2010

Note to instructor: Check PIRS to ensure this outline is the most current version.